Statistics

Spring 2023

Lecture 12

Feb 19-8:47 AM

Statistic → Describe Sample

Parameter → Describe Population

we use Statistic to estimate Parameters.

Estimation is a range of Values that is called Confidence Interval.

Every Confidence interval comes with some Confidence level.

Confidence level is $(1-\alpha)\cdot 100\%$

$0<\alpha<1$, α is called Significance level.

Alpha

If $\alpha=.1 \rightarrow (1-\alpha)\cdot 100\% = (1-.1)\cdot 100\% = 90\%$

C-level

If $\alpha=.02 \rightarrow (1-\alpha)\cdot 100\% = (1-.02)\cdot 100\% = 98\%$

when α not given ⇒ use .05

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$\alpha \to$ Significance level

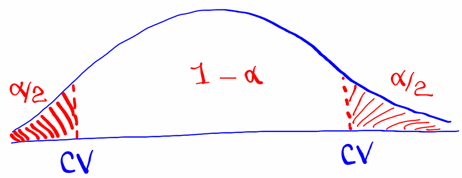
$0 < \alpha < 1$

$(1-\alpha)\cdot 100\% \to$ Confidence level (C-level)

$1-\alpha$ is the middle area of the graph of the Prob. dist.

Area on each tail of the graph of the Prob. dist. is $\alpha/2$.

Values that Separate the middle Area and tails are Called Critical Values (C.V.)



If α not given $\Rightarrow$ Use .05

If C-level not given $\Rightarrow$ Use 95%

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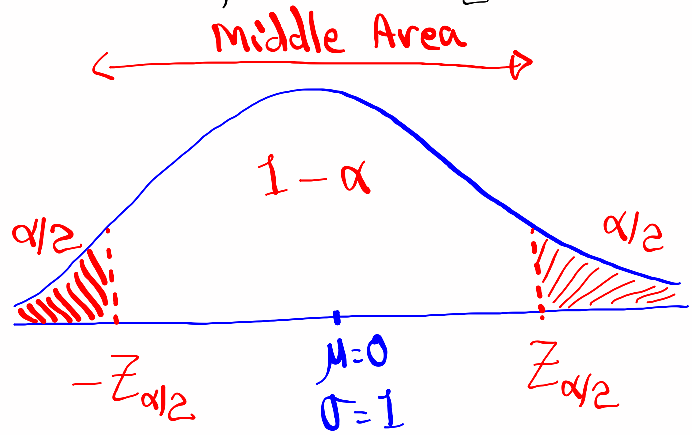
$Z_{\alpha/2}$ is a Value, Rounded To 3-decimal, that Separates the right area $\alpha/2$ from the rest.



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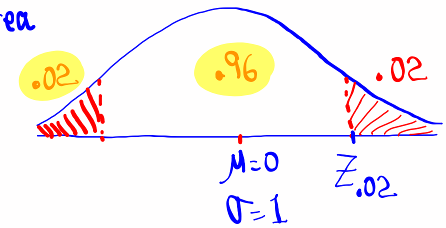
find $Z_{.02}$ ← Right area

$\frac{\alpha}{2} = .02$

$\alpha = .04$

$1-\alpha = .96$

C-level $= 96\%$



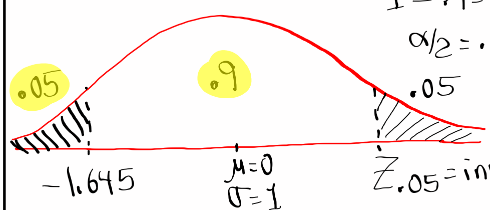
$Z_{.02} = \text{invNorm}(.98, 0, 1) = \boxed{2.054}$

find $Z_{\alpha/2}$ for $\underbrace{90\% \text{ C-level.}}$

middle area $= .9$

$1 - .9 = .1 \leftarrow \alpha$

$\alpha/2 = .05$



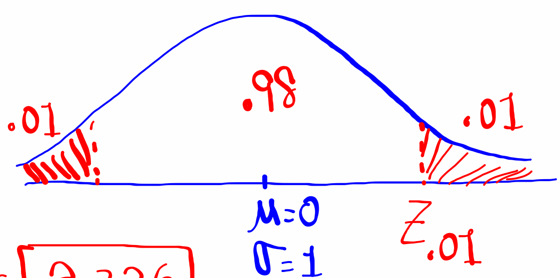
$Z_{.05} = \text{invNorm}(.95, 0, 1) = \boxed{1.645}$

May 2-7:06 PM

find $Z_{.01}$

Right area $\rightarrow \alpha/2 = .01 \rightarrow \alpha = .02$

$1-\alpha = .98 \leftarrow$ Middle area



$Z_{.01} = \text{invNorm}(.99, 0, 1) = \boxed{2.326}$

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Estimating $\underbrace{\text{Population Proportion:}}_{p}$

Final Answer $\quad < p < \quad$ this range of Values is called Confidence Interval.

$\hat{p} - E < p < \hat{p} + E$ ← margin of error

p-hat, Sample Proportion "Point-estimate" ← Best Guess

Suppose $\hat{p} = .42$, and $E = .05$

Conf. Interval $\hat{p} - E < p < \hat{p} + E$

$.42 - .05 < p < .42 + .05$ → $\boxed{.37 < p < .47}$

May 2-7:15 PM

Suppose $\boxed{15\% \text{ of a Sample}}$ of students were left-handed. $\hat{p} = .15 \quad E = .06$

Use $\boxed{\text{margin of error } 6\%}$ to find confidence interval for the prop. of all students that are left-handed.

$\hat{p} - E < p < \hat{p} + E$

$.15 - .06 < p < .15 + .06$

$.09 < p < .21$

$9\% < p < 21\%$

With some confidence level, we say $\boxed{\text{Prop. of all}}$ students that are left-handed is between 9% and 21%.

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How to find $\hat{p}$:

$\hat{p} \to$ Sample Proportion

$\hat{p} = \frac{x}{n}$ ← # of favorable responses ; ← Sample size

$\hat{p} = \frac{x}{n}$, Always round to 3-decimal places

ex: I surveyed 100 students, and 18 were smokers.

$n = 100$, $x = 18$, $\hat{p} = \frac{x}{n} = \frac{18}{100} = \boxed{.18}$

$\hat{q} = 1 - \hat{p} = 1 - .18 = \boxed{.82}$

18% were smokers
82% were not smokers.
} from survey.

May 2-7:26 PM

I surveyed 400 adults and 125 of them were in favor of abortion.

1) $n = 400$

2) $x = 125$

3) $\hat{p} = \frac{x}{n} = \frac{125}{400} = .313$

$\boxed{\hat{p} \approx 31\%}$

4) $\hat{q} = 1 - \hat{p} = .687$ $\boxed{\hat{q} \approx 69\%}$

Use margin of error of 5%, find conf. interval for the prop. of all adults that are in favor of abortion.

$\hat{p} - E < p < \hat{p} + E$

$.313 - .05 < p < .313 + .05$

$.308 < p < .363$ $\boxed{31\% < p < 36\%}$

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How to find margin of error E:

$$E = Z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}\,\hat{q}}{n}}$$

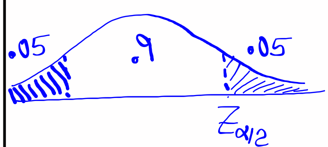
$Z_{\alpha/2}$ → Critical value for $(1-\alpha)\cdot 100\%$

C-level.

Given $n=100$, $x=80$, C-level: .9

$\hat{p} = \frac{x}{n} = .8$, $\hat{q} = 1-\hat{p} = .2$

$E = Z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}\,\hat{q}}{n}} = 1.645\sqrt{\frac{(.8)(.2)}{100}} \approx \boxed{.066} \approx 7\%$



$\hat{p} - E < p < \hat{p} + E$

$.8 - .066 < p < .8 + .066$

$.734 < p < .866$

$\boxed{73\% < p < 87\%}$

$Z_{\alpha/2} = \text{invNorm}(.95, 0, 1) = \boxed{1.645}$

we are 90% confident that pop. prop. is between 73% & 87%..

May 2-7:37 PM

In a survey of 250 randomly selected students, 76% of them had iphone.

$n = 250$ $\hat{p} = .76$ → $\hat{q} = 1-\hat{p} = .24$

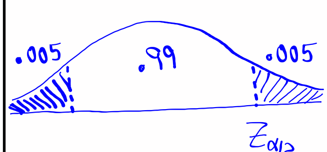
$x = n\hat{p}$
$= 250(.76)$
$= \boxed{190}$

Find 99% conf. interval for the prop. of all students that have iPhone.

C-level: .99

$\hat{p} - E < p < \hat{p} + E$

$.76 - .070 < p < .76 + .070$



$.69 < p < .83$

$69\% < p < 83\%$

we are 99% confident that between 69% & 83% of all students have iPhone.

$Z_{\alpha/2} = \text{invNorm}(.995, 0, 1) = \boxed{2.576}$

$E = Z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}\,\hat{q}}{n}}$

$= 2.576 \cdot \sqrt{\frac{(.76)(.24)}{250}}$

$\approx \boxed{.070}$

May 2-7:46 PM

Using TI Command

STAT → TESTS ↓ 1-PropZInt

x: 190

n: 250

C-level: .99

Calculate

(.69042, .82958)

.690 < p < .830

$\boxed{69\% < p < 83\%}$

$E = \dfrac{.830 - .690}{2} = \boxed{.07}$

$\hat{p} = \dfrac{.830 + .690}{2} = \boxed{.76}$

May 2-7:56 PM

46% of 185 randomly selected students were in support of online classes with Zoom meeting.

$n = 185$

$\hat{p} = .46$

$\Rightarrow x = n\hat{p} = 185(.46) = 85.1$ $\boxed{x = 86}$

if decimal, round-up

Find conf. interval for the prop. of all students in support of online classes with Zoom meeting.

→ No C-level

⇒ Use 95%

$\boxed{.393 < p < .537}$

1-PropZInt

39% < p < 54%

$E = \dfrac{.537 - .393}{2} = \boxed{.072}$

$\hat{p} = \dfrac{.537 + .393}{2} = \boxed{.465}$

May 2-8:11 PM

Estimating $\underbrace{\text{Population Mean}}_{\mu}$:

Conf. interval $\bar{x} - E < \mu < \bar{x} + E$

Sample Mean "point-estimate"

Margin of error

Case I: σ Known	
$E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$ $(1-\alpha)\cdot 100\%$ C-level	
TI Command Z Interval inpt: $\boxed{\text{Stats}}$	

May 2-8:19 PM

Given $n=32$, $\bar{x}=80$, $\sigma=10$, C-level: 90%

$\boxed{\text{STAT}}$ $\boxed{\rightarrow}$ TESTS $\boxed{\downarrow}$ $\boxed{\text{Z Interval}}$

inpt: $\boxed{\text{Stats}}$

$\sigma=10$

$\bar{x}=80$

$n=32$

C-level: .9

$\boxed{\text{Calculate}}$

(77.092, 82.908)

$77.092 < \mu < 82.908$

Since $\bar{x}$ is a whole #,

$\boxed{77 < \mu < 83}$

$E = \frac{83-77}{2} = \boxed{3}$

$\bar{x} = \frac{83+77}{2} = \boxed{80}$

May 2-8:23 PM

20 randomly selected students had a mean age of 30.8 yrs.

$n = 20$, $\bar{x} = 30.8$

$\sigma = 12.5$

It is known that standard deviation of ages of all students is 12.5 yrs.

C-level: .98

Find 98% conf. interval for the mean age of all students.

$< \mu <$

Since σ is known → use ZInterval

inpt: $\boxed{\text{Stats}}$

$\sigma = 12.5$

$\bar{x} = 30.8$

$n = 20$

C-level: .98

$\boxed{\text{Calculate}}$

(24.298, 37.302)

Since $\bar{x}$ is in 1-decimal

$\boxed{24.3 < \mu < 37.3}$

$E = \frac{37.3 - 24.3}{2} = \boxed{6.5}$

$\bar{x} = \frac{37.3 + 24.3}{2} = \boxed{30.8}$

May 2-8:28 PM

Estimating $\underbrace{\text{Population Mean}}_{\mu}$:

Conf. interval $\bar{x} - E < \mu < \bar{x} + E$

$\bar{x}$: Sample Mean "point-estimate"

E: Margin of error

Case I: σ Known	Case II: σ Unknown
$E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$ $z_{\alpha/2}$: $(1-\alpha) \cdot 100\%$ C-level	$E = t_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$ $t_{\alpha/2}$: $(1-\alpha) \cdot 100\%$ C-level $df = n - 1$
TI Command: ZInterval inpt: $\boxed{\text{Stats}}$	TI Command: TInterval inpt: $\boxed{\text{Stats}}$

May 2-8:19 PM

Given: $n = 10$, $\bar{x} = 84$, $S = 12$, C-level: 90%

Find conf. interval for μ.

Since σ unknown

$\Rightarrow$ Use TInterval

inpt: $\boxed{\text{Stats}}$

$\bar{x} = 84$

$S = 12$

$n = 10$

C-level: .9

$\boxed{\text{Calculate}}$

(77.044, 90.956)

Since $\bar{x}$ is a whole #

$\boxed{77 < \mu < 91}$

$E = \frac{91-77}{2} = \boxed{7}$

$\bar{x} = \frac{91+77}{2} = \boxed{84}$

May 2-8:40 PM

15 randomly Selected Students had a mean age of 31.4 and Standard deviation of 9.5.

$n = 15$, $\bar{x} = 31.4$, $S = 9.5$

No C-level $\Rightarrow$ .95

Find Conf. interval for the mean age of all Students.

$< \mu <$

Since σ is unknown

$\Rightarrow$ Use TInterval

inpt: $\boxed{\text{Stats}}$

$\bar{x} = 31.4$

$S = 9.5$

$n = 15$

C-level: .95

$\boxed{\text{Calculate}}$

(26.139, 36.661)

Since $\bar{x}$ is in 1-decimal

$\boxed{26.1 < \mu < 36.7}$

$E = \frac{\quad - \quad}{2}$

$\bar{x} = \frac{\quad + \quad}{2}$

May 2-8:44 PM

I randomly Selected 12 exams, here are the Scores:

75	82	98	100
68	55	72	90
58	77	90	95

1) Find $\bar{x}$ & S.
Round to whole #

$\bar{x} = 80$

$S = 15$

2) Find 99% Conf. interval for the mean Score of all exams.

σ not given $\Rightarrow$ Use TInterval

inpt: Stats

$\bar{x}=80$
$S=15$
$n=12$
C-level: .99

(66.551, 93.449)

Since $\bar{x}$ is a whole #

$\boxed{67 < \mu < 93}$

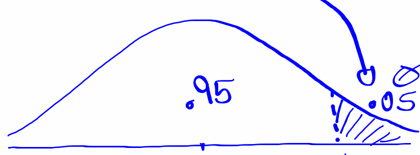
$E = \dfrac{93-67}{2} = \boxed{13}$

$\bar{x} = \dfrac{93+67}{2} = \boxed{80}$

May 2-8:52 PM

Find $t_{\alpha/2}$ for $\alpha = .1$ with $df = 8$.

$\alpha/2 = .05$



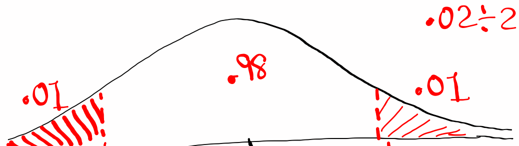
$\mu = 0$
σ unknown
$df = 8$

$t_{.05} = invT(.95, 8) = \boxed{1.860}$

Find $\pm t_{\alpha/2}$ for 98% C-level with $df = 12$.

$1 - .98 = .02$

$.02 \div 2 = .01$



$-t_{.01}$

$\mu = 0$
σ unknown
$df = 12$

$t_{.01} = invT(.99, 12) = \boxed{2.681}$

May 2-9:04 PM

what is degrees of freedom?

10 people in a meeting.

10 Donuts on the table.

First Person has 10 choices.

Second " " 9 choices

third " " 8 "

⋮

Last person has no choice (1 Donut)

$df = 9$ 9 people choices.

May 2-9:09 PM

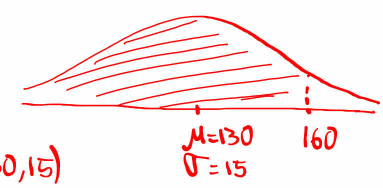
Class QZ 5

Given $N(130, 15)$

1) Find $P(x < 160)$

$= \text{normalcdf}(-E99, 160, 130, 15)$

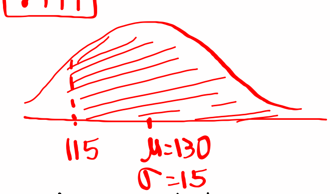
$= \boxed{.977}$



2) Find $P(x > 115)$

$= \text{normalcdf}(115, E99, 130, 15)$

$= \boxed{.841}$

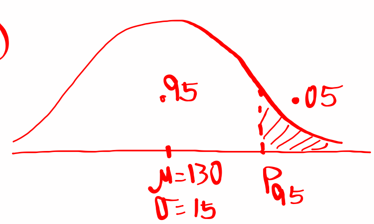


3) Find $x = P_{95}$, Round to whole #.

$x = \text{invNorm}(.95, 130, 15)$

$= 154.673$

$\approx \boxed{155}$



May 2-9:12 PM